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NONLINEAR NONRADIAL OSCILLATIONS SELF-GRAVITATING DISK WITH ACCOUNTING HALO

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Annotatsiya. *Ushbu maqolada galoni hisobga olgan holda galaktikalar disk osti tuzilmalari evolutsiyasining muommosini ko'rib chiqdik. Nochiziqli noradial tebranuvchi diskning o'z tekisligidagi evolutsiyasi qoramtir materiya galosining asosiy parametrlariga bog'liqligini sonli usul yordamida o'rgandik. Qoramtir materiya galosi disk tekisligidagi beqarorlikni barqarorlaydi, vertikal tebranishlarni esa beqarorlaydi. Global diskning tuzilishi qoramtir materiya galosining massasi va shakliga juda bog'liq. O'zgravitatsiyalanuvchi diskning tebranish jarayoni qoramtir materiya galosining parametrlariga evolutsion bog'liqlik tenglamalari tuzilgan.*

Kalit so'zlari: *spiral galaktikalar, galo, qoramtir materiya galosi, nochiziqli noradial tebranishlar*

Аннотация: *В данной работе мы рассматриваем проблему эволюции дисковой подсистемы галактик с учетом гало. Для этого мы изучили зависимость эволюция нелинейно нерадиально колеблющегося в своей плоскости диска в зависимости от основных параметров гало темной материи численно. Гало темной материи стабилизирует нестабильности в плоскости диска, но дестабилизирует его вертикальные колебания. Структура глобального диска сильно зависит от массы и формы гало темной материи. Построены эволюционные зависимости процесса колебаний самогравитирующего диска от указанных параметров гало темной материи.*

Ключевые слова: *спиральные галактики, гало, гало темной материи, нелинейно-нерадиалные колебания*

Abstract: *In this paper, we consider the problem of the evolution of the disk subsystem of galaxies, taking into account the halo. To do this, we studied the dependence of the evolution of a non-linear non-radially oscillating disk in its plane on the main parameters of the dark matter halo numerically. The dark matter halo stabilizes instabilities in the disk plane but destabilizes its vertical oscillations. The structure of the global disk strongly depends on the mass and shape of the dark matter halo. Evolutionary dependences of the process of oscillations of a self-gravitating disk on the indicated parameters of the dark matter halo are constructed.*

Keywords: *spiral galaxies, halo, dark matter halo, non-linear non-radial oscillations*

This work is devoted to modeling and studying the nonlinear evolution of two-dimensional models. The well-studied equilibrium two-dimensional models of Bistavatoy-Kogan and Zel'dovich [29, 30] were taken as initial ones. Let us consider perturbations of a special class [31] under which: firstly, the spatial density remains homogeneous, and secondly, the system in the perturbed state retains an elliptical shape. Then, the relationship between the perturbed and unperturbed coordinates and velocities r0, v0, and r, v can be described using the generating function

$$w(r, v_0, t) = \frac{1}{2} r M(t) r + r N(t) v_0 + \frac{1}{2} v_0 P(t) v_0 \qquad (1)$$

where *M, N, P* – two-dimensional matrices, M and P are symmetric, then

$$v = grad_r w = Mr + Nv \qquad (2)$$

$$r_0 = grad_v w = Nr + Pv \qquad (3)$$

here N is the transposed matrix. Previously, in [32, 33], using the Hamilton-Jacobi equations, the equations for the nonlinear evolution of the disk and cylindrical models in a perturbed state were obtained in matrix form. It turned out that these equations can be obtained directly by differentiating equations (2) and (3) and comparing the coefficients at the same terms. A detailed derivation can be found in [34,35]. For elliptical cylinder

$$\begin{cases} \frac{dP}{dt} + NN = 0 \\ \frac{dN}{dt} + MN = 0 \\ \frac{dM}{dt} + M^2 + \frac{2K_1^{\frac{1}{2}}}{Tr\left(K_1^{-\frac{1}{2}}\right)} \end{cases} \qquad (4)$$

where K_1 - two-dimensional matrix associated with the potential of the cylindrical model

$$\varphi_{c\,ln\,dr} = \frac{rK_1^{\frac{1}{2}}}{Tr\left(K_1^{\frac{-1}{2}}\right)} \qquad (5)$$

in a coordinate system oriented along the main axes of the model

$$K_1 = \begin{vmatrix} a^{-2} & 0 \\ 0 & b^{-2} \end{vmatrix} \qquad (6)$$

where *a* and *b* – major and minor semiaxis of model. In [36], the dependence of K_1 on the matrices M, N, P

$$K_1 = N[E + P^2 + \Omega(PJ - JP)]^{-1} N^* \qquad (7)$$

here *E* – identity matrix, $J = \begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix}$, Ω - the ratio of centroid speed to circular speed at the same point. For a disk-shaped model, the last of equations (2.4) should be replaced by the following

$$\frac{dM}{dt} + M^2 + 2K_2 = 0 \qquad (8)$$

where K_2 – a two-dimensional matrix that determines the disk potential

$$\varphi_{disk} = -r \cdot K_2 r \qquad (9)$$

In a coordinate system oriented along the main axes of the disk

$$K_2 = \begin{vmatrix} \alpha & 0 \\ 0 & \beta \end{vmatrix} \qquad (10)$$

where

$$\alpha = \frac{2}{\pi k^2 a^3}\left[\widetilde{K}(k) - \tilde{E}(k)\right] \quad \beta = \frac{2}{\pi k^2 a^3}\left[\frac{a^2}{b^2}\widetilde{K}(k) - \tilde{E}(k)\right] \qquad (11)$$

parametr $k^2 = \frac{(a^2-b^2)}{a^2}$ characterizes compression, $\widetilde{K}(k)$ and $\tilde{E}(k)$ - complete elliptic integrals of the 1st and 2nd order.

Since the elements of the matrices M, N, P are discontinuous functions of time (even in the stationary state, when $M_0 = tg\ t\ E,\ N_0 = sec\, t\, E,\ P_0 = -tg\ t\ E$, to perform numerical integration, transform system (4) to this form, where all new variables would be continuous functions of time, moreover, all construction elements on the right-hand side would also be continuous. To this end, we introduce new matrices related to M, N, and P by the following relations (see [35]):

$$U = N^{-1}; \quad S = PN^{-1}; \quad T = N^{-1}M; \quad S_1 = \frac{dS}{dt} \qquad (12)$$

In the stationary state for the matrices U, S, T we get

$$U = N_0^{-1} = cos\, t \cdot E, \quad S_0 = P_0 N_0^{-1} = -\,sin\, t \cdot E, \quad T_0 = N_0^{-1} M_0$$

To check the continuity of the elements of the matrix S_1, we express it in terms of M, N, P

$$S_1 = \frac{dS}{dt} = \frac{d(PN^{-1})}{dt} = \frac{dP}{dt}N^{-1} - PN^{-1}\frac{dN}{dt}N^{-1}$$

having (2.4), after minor transformations, we get

$$S_1 = N^* + PN^{-1}M = -N^* + PT$$

Whence it follows

$$S_{1_0} = -\,cos\, t \cdot E$$

We now differentiate expressions (12), taking into account (4)

$$\frac{dU}{dt} = -N^{-1}\frac{dN}{dt}N^{-1} = -N^{-1}(-MN)N^{-1} = T,$$

$$\frac{dT}{dt} = -N^{-1}\frac{dN}{dt}N^{-1}M + N^{-1}\frac{dM}{dt} = N^{-1}MNN^{-1}M + N^{-1}\left[-M^2 - \frac{2K_1^{\frac{1}{2}}}{Tr\left(K_1^{-\frac{1}{2}}\right)}\right] =$$

$$= -\frac{2N^{-1}K_1^{\frac{1}{2}}}{Tr\left(K_1^{-\frac{1}{2}}\right)} = \frac{2UK_1^{\frac{1}{2}}}{Tr\left(K_1^{-\frac{1}{2}}\right)}.$$

$$\frac{dS_1}{dt} = \frac{d}{dt}(PT - N^*) = \frac{dP}{dt}T + P\frac{dT}{dt} - \frac{dN^*}{dt} = -N^*NT - \frac{2PN^{-1}K_1^{\frac{1}{2}}}{Tr\left(K_1^{-\frac{1}{2}}\right)} + N^*M =$$

$$= -N^*NN^{-1}M - \frac{2SK_1^{\frac{1}{2}}}{Tr\left(K_1^{-\frac{1}{2}}\right)} + N^*M = -\frac{2SK_1^{\frac{1}{2}}}{Tr\left(K_1^{-\frac{1}{2}}\right)}$$

Thus, the system of equations (4) describing the nonlinear evolution of the cylindrical model is transformed into the following form

$$\begin{Bmatrix} \frac{dU}{dt} = T; & \frac{dS}{dt} = S_1; \\ \frac{dT}{dt} = \frac{2UK_1^{\frac{1}{2}}}{Tr\left(K_1^{-\frac{1}{2}}\right)}; & \frac{dS_1}{dt} = -\frac{2SK_1^{\frac{1}{2}}}{Tr\left(K_1^{-\frac{1}{2}}\right)}; \end{Bmatrix} \qquad (13)$$

where K_1 in new notation

$$K_1 = [U^*U + S^*S + \Omega(S^*JU - U^*JS)]^{-1} \qquad (14)$$

for the disk model, similar equations are obtained

$$\frac{dU}{dt} = T; \quad \frac{dT}{dt} = -2UK_2; \quad \frac{dS}{dt} = S_1; \quad \frac{dS_1}{dt} = -2SK_2 \qquad (15)$$

Instead of the functional dependence of K_2 on U, T, S, and S_1, it is more convenient to determine the dependence on K_2 (K_1).

By introducing the parameter $p = M_{galo}/\mathrm{M}_{disk}$ as the ratio of the masses of the halo and the disk, we have obtained a system of disk evolution equations that will look like this

$$\frac{dU}{dt} = T, \quad \frac{dT}{dt} = -2UK_2 - pA_1U, \quad \frac{dS}{dt} = D, \quad \frac{dD}{dt} = -2SK_2 - pA_1S \qquad (43)$$

To integrate this system of matrix differential equations, we set the initial conditions. We introduce the matrix

$$H = \begin{pmatrix} \mu & 0 \\ 0 & \mu^{-1} \end{pmatrix}, \qquad (44)$$

that

$$\vec{r} = H\vec{r}_0 \quad \text{и} \quad \vec{v} = H^{-1}\vec{v}_0.$$

Thus, the value of μ characterizes non-linear deviations from the unperturbed equilibrium state, for which μ=1. Then for a moment

$$t_0 = 0 \qquad (N_1)_0 \equiv 0, \qquad (N_2)_0 \equiv H^{-1} \qquad (N_3)_0 \equiv 0.$$

Since we were also interested in considering instabilities against the background of radially oscillating models, it would be expedient here to also present calculations regarding the behavior of the considered nonlinear model (35).

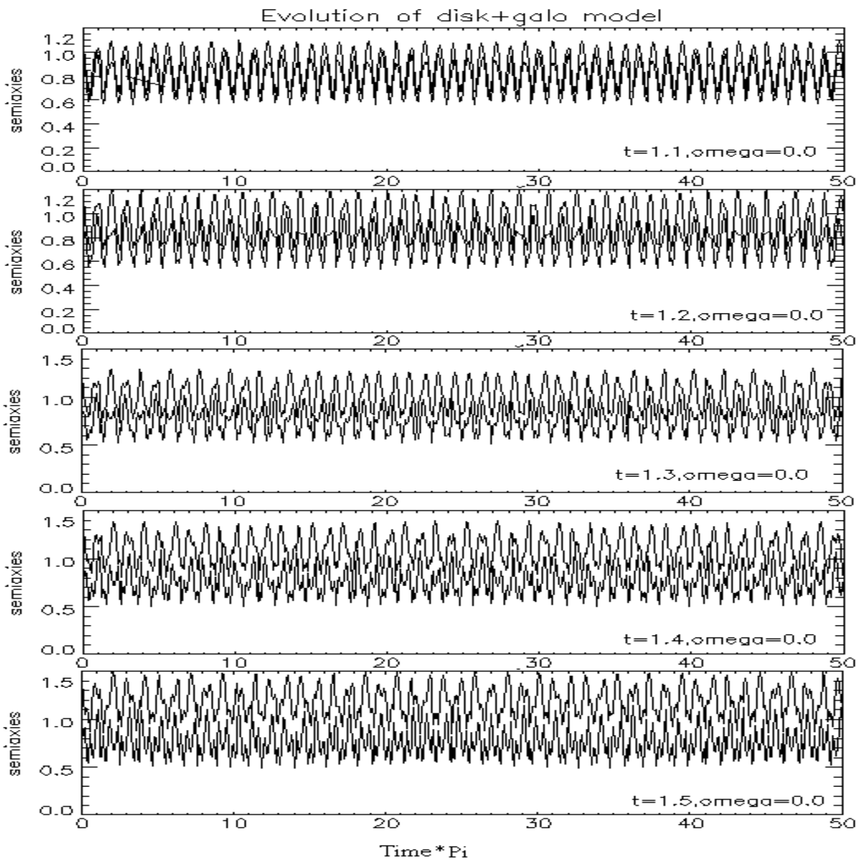

FIG. 12. Behavior of the semi-axes of the disk taking into account the halo in the time interval from 0 to 50π for different initial perturbations.

To this end, we have numerically solved the corresponding Cauchy problem with given initial conditions. Calculations of the evolution of the disk model were carried out on the time interval [0; 50π] with relative integration accuracy $\Delta=10^{-7}$. The values a and b were printed at intermediate points $t_i = \frac{\pi i}{20}$.

Conclusion.

1. A method has been developed for the numerical analysis of the system of equations for the evolution of a non-linear non-radial oscillation of a self-gravitating disk, taking into account the halo.
2. The dependences of the major and minor semiaxes of the disk on time are obtained for various values of the system parameters.
3. The critical values of q and the disk rotation parameter Ω are determined, at which the halo stabilizes non-linear non-radial oscillations of the disk subsystem of galaxies.
4. It is proved that for large μ the curves of the disk semiaxes behave identically and starting from μ=1.15 the oscillations of the disk model become unstable and for μ≥1.8 the disk semiaxes grow linearly with time and the disk breaks up, i.e. dissipates.

5. It has been established that at P=1 and $0 \leq \Omega \leq 1$ the halo stabilizes non-linear non-radial oscillations of the disk, but starting from the values $P \leq 0.001$ it ceases to stabilize these oscillations

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